(1) A number is <u>rational</u> if it can be expressed in the form $\frac{m}{n}$ where m and n are integers with $n \neq 0$. An irrational is a real number which is not rational. Prove that the following statements are true when a, b, c and d are rational numbers.

(i) If $b \neq 0$ then $a - b$ and a/b are rational.

(ii) If ξ is an irrational and $a + b\xi = c + d\xi$ then $a = c$ and $b = d$.

(iii) If $\sqrt{a+\sqrt{b}} + \sqrt{a-\sqrt{b}}$ is rational, then both of the numbers $2(a + \sqrt{a^2 - b})$ and $a^2 - b$ are squares of rational numbers.

(2) Evaluate $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \cdots + \frac{1}{4n^2 - 1}$.

(3) Sketch the graph of $y^2 = x^3 - x$.

(4) Prove the following inequalities, where x, y and z are arbitrary positive real numbers:

$$x^2 + y^2 + z^2 \geq xy + yz + zx;$$

(5) The letters a, b, c, d denote four different positive integers, none of which is equal to 1. Consider the following statements.

(i) $b = a + 7$ and $b < c < d$.

(ii) $ab = cd$.

(iii) a and b are both prime.

(iv) b divides a.

Show that there is only one pair of statements which can be true simultaneously.

(6) Sum the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

(7) Sketch the curve $y^2 = \cos x$ for $-2\pi \leq x \leq 2\pi$.

(8) Simplify $\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\cdots\left(1 - \frac{1}{n^2}\right)$.